Electronic supplementary material for

The evolution of anti-social rewarding and its counter-measures in public goods games

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SI Appendix Sections 1 to 6

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Appendix

Here, I first determine, in Section 1, the expected payoffs for public goods games (PGG) with reward funds and provide analytical investigation of game dynamics for the basic model. I further investigate variants of reward funds. I consider, in Section 2, different rewarding abilities for contributors and non-contributors to the PGG, then exclusion from reward funds in Section 3, and, in Section 4, rewards dependent on the PGG’s success. In Section 5, I investigate with numerical simulations the dynamics in the interior of the simplex $S_4$ for each scenario, and for a large range of parameter values. Finally, in Section 6 I investigate the variant with reward funds dependent on the PGG’s production where $\beta$ is an evolvable cultural trait.

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**Section 1: Basic model.** I consider the replicator dynamics for the defectors (D), rewarding defectors (RD), cooperators (C) and rewarding cooperators (RC), with frequencies $x, y, z,$ and $w$ respectively. Thus, $x, y, z, w \geq 0$ and $x + y + z + w = 1$. I denote the expected payoff for the four strategies by $P_S$, with $S = D, \text{RD}, C$ or RC. The evolutionary fate of the population can be modeled by the replicator dynamics:

$$
\dot{x} = x(P_D - \bar{P}),\dot{y} = y(P_{RD} - \bar{P}),\dot{z} = z(P_C - \bar{P}),\dot{w} = w(P_{RC} - \bar{P}).
$$

[S1]

where $\bar{P} := xP_D + yP_{RD} + zP_C + wP_{RC}$ describes the average payoff in the entire population.

I define now the expected payoffs for each of the four strategies. In the PGG stage of the game, everybody gets an equal share of the public good produced by the total number of contributors C and RC (with frequencies $z$ and $w$, respectively) among the focal player’s $N - 1$ coplayers. Therefore, the expected payoff for the PGG for defectors D and RD is

$$
P_{D1} = P_{RD1} = \frac{r_1c_1}{N} (N - 1)(z + w).
$$

[S2]

Contributors receive in addition their own share but pay the cost of contribution, therefore their payoff is reduced by $\sigma = c_1 \left(1 - \frac{r_1}{N}\right)$, which gives

$$
P_{C1} = P_{RC1} = \frac{r_1c_1}{N} (N - 1)(z + w) - \sigma.
$$

[S3]

For the prosocial reward fund, a focal cooperator C in a group with $N_C$ (i.e. C and RC) contributors (focal included) and $N_{RC}$ other rewarding cooperators RC, will receive a
reward of \( r_2 c_2 N_R / N_C \) \((0 \leq N_R \leq N_C - 1)\). Therefore, the expected reward for a cooperator C in a group with \( N_C \) contributors is given by

\[
P_{C2}(N_C) = \sum_{N_R=0}^{N_C-1} \binom{N_C - 1}{N_R} \left( \frac{w}{z+w} \right)^{N_R} \left( \frac{z}{z+w} \right)^{N_C-N_R-1} \frac{r_2 c_2 N_R}{N_C}
\]

\[= r_2 c_2 \left( 1 - \frac{1}{N_C} \right) \left( \frac{w}{z+w} \right). \tag{S4}\]

where

\[
\binom{N_C - 1}{N_R} \left( \frac{w}{z+w} \right)^{N_R} \left( \frac{z}{z+w} \right)^{N_C-N_R-1}
\]

is the probability of having \( N_R \) rewarders and \( N_C - N_R - 1 \) non rewarding cooperators in a group of \( N_C \) contributors. Hence, the expected reward for a cooperator C is

\[
P_{C2} = \sum_{N_C=1}^{N} \binom{N - 1}{N_C - 1} (z+w)^{N_C-1} (x+y)^{N-N_C} P_{C2}(N_C)
\]

\[= r_2 c_2 \left( 1 - \frac{1}{N} \right) \left( \frac{x+y}{N(z+w)} \right) \left( \frac{w}{z+w} \right). \tag{S5}\]

In a group of \( N_C \) contributors, switching from R to C yields \( c_2 (1 - r_2 / N_C) \). Hence, the expected reward for a rewarding cooperator \( P_{RC2} \) is reduced from \( P_{C2} \) by

\[
c_2 \sum_{N_C=1}^{N} \binom{N - 1}{N_C - 1} (z+w)^{N_C-1} (x+y)^{N-N_C} \left( 1 - \frac{r_2}{N_C} \right) = c_2 \left( 1 - r_2 \frac{1-(x+y)^N}{N} \right)
\]

\[= c_2 \left( 1 - \frac{r_2}{N} \frac{1-(x+y)^N}{z+w} \right) := F_C(z+w). \tag{S6}\]
The expected reward for defectors and rewarding defectors can be calculated in a similar way. Hence the expected reward for a defector D is given by

\[ P_{D2} = r_2 c_2 \left( 1 - \frac{1 - (z + w)^N}{N(x + y)} \right) \left( \frac{y}{x + y} \right). \]  

[S7]

and the reward for RD players \( P_{RD2} \) is reduced from \( P_{D2} \) by

\[ c_2 \left( 1 - \frac{r_2}{N} \frac{1 - (z + w)^N}{x + y} \right) = c_2 \left( 1 - \frac{r_2}{N} \frac{1 - (z + w)^N}{1 - (z + w)} \right) := F_D(z + w). \]  

[S8]

\( F_C(z + w) \) has one root in the open interval (0,1) if, and only if, \( 1 < r_2 < N \) since \( F_C(z + w) \) is monotonic, \( F_C(0) = c_2(1 - r_2/N) > 0 \), and \( F_C(1) = c_2(1 - r_2) < 0 \). \( F_D(z + w) \) also has one root in the open interval (0,1) if, and only if, \( 1 < r_2 < N \) since \( F_D(z + w) \) is monotonic, \( F_D(1) = c_2(1 - r_2/N) > 0 \), and \( F_D(0) = c_2(1 - r_2) < 0 \). Consequently, the advantage non-rewarding players have over their corresponding rewarding type decreases and changes from positive to negative as the frequency of the opposite type (i.e. \( z + w \) for D and \( x + y \) for C) increases.

Note that since \( F_C(z + w) = F_D(1 - (z + w)) \), both functions are symmetric about the vertical line \( z + w = x + y = 0.5 \). Consequently, the only point where both RD and RC can have similar payoffs to D and C, respectively, will be when \( z + w = 0.5 \). Thus, the only interior equilibria between the four strategies that could exist will be when \( z + w = 0.5 \).

We can now obtain the total expected payoffs for all strategies: \( P_D = P_{D1} + P_{D2} \), \( P_{RD} = P_{RD1} + P_{RD2} \), \( P_C = P_{C1} + P_{C2} \), \( P_{RC} = P_{RC1} + P_{RC2} \). Consequently, the average payoff in the population is

\[ \bar{P} = c_1(r_1 - 1)(z + w) + c_2(r_2 - 1)(y + w). \]  

[S9]
The dynamics of the four strategies is represented in the state space $S_4 = \{(x, y, z, w) : x, y, z, w \geq 0, x + y + z + w = 1\}$, where the four homogeneous states $D (x = 1)$, $RD (y = 1)$, $C (z = 1)$ and $RC (w = 1)$, are trivial equilibria.

I present now the detailed dynamics on each of the edges of $S_4$. On the edge C-D: $y + w = 0$ and on the edge RC-RD: $x + z = 0$, resulting in $\dot{x} = (P_D - P_C)x(1 - x) = \sigma x(1 - x) > 0$ and $\dot{y} = (P_{RD} - P_{RC})y(1 - y) = \sigma y(1 - y) > 0$, respectively, where $\sigma = c_1(1 - r_1/N)$. Therefore, the direction of the dynamics goes from C to D and from RC to RD, respectively.

On the edge RD-D: $z + w = 0$, resulting in $\dot{z} = (P_D - P_{RD})z(1 - z) = c_2 \left(1 - \frac{r_2}{N}\right)z(1 - z) > 0$. Therefore, the direction of the evolution on this edge goes from RD to D.

On the edge RC-C: $x + y = 0$, resulting in $\dot{z} = (P_D - P_{RD})z(1 - z) = c_2 \left(1 - \frac{r_2}{N}\right)z(1 - z) > 0$. Therefore, the direction of the evolution on this edge goes from RC to C.

On the edge C-RD: $x + w = 0$, resulting in $\dot{y} = (P_{RD} - P_C)y(1 - y) = [c_2(r_2 - 1) + \sigma]y(1 - y) > 0$. Therefore, the direction of the dynamics goes from C to RD.

On the edge D-RC: $y + z = 0$ resulting in $\dot{w} = (P_{RC} - P_D)w(1 - w) = [c_2(r_2 - 1) - \sigma]w(1 - w)$. Therefore, the direction of the dynamics on this edge will depend on whether the net reward $c_2(r_2 - 1)$ can compensate for the net cost $\sigma$ of the PGG. If $c_2(r_2 - 1) - \sigma < 0$, the direction of the evolution will go from RC to D. In the boundary case that $c_2(r_2 - 1) - \sigma = 0$, the edge becomes a line of fixed points. In order to determine the stability conditions of the line, one can solve $c_2(r_2 - 1) - \sigma = 0$ for $c_2$ which yields

$$c_2 = \frac{\sigma}{r_2 - 1}$$
Then, by substituting equation (S10) into $P_D, P_{RD}, P_C$ and $P_{RC}$, one can determine that the line will be stable against C if

$$M_C(w) := -r_2[1 - (1 - w)^N] + Nw < 0 \quad [S11]$$

and against RD if

$$M_{RD}(w) := -r_2(1 - w^N) + N(1 - w) > 0. \quad [S12]$$

Since $M_C(0) = 0, M_C'(0) = N - r_2 > 0$ and $M_C'(0) = N(1 - r_2) < 0$ the function has one root in the open interval $(0,1)$. Similarly, $M_{RD}(1) = 0, M_{RD}(0) = r_2 - N < 0$ and $M_{RD}'(1) = N(1 - r_2) < 0$ the function also has one root in the open interval $(0,1)$. If both $M_C(w)$ and $M_{RD}(w) < 0$, the line is separated into a stable segment against both RD and C $(0 \leq w \leq w_{RC})$ and an unstable one $(w_{RC} < w \leq 1)$. In addition, since $M_C(1/2 - w) = -M_{RD}(1/2 + w)$, both functions are symmetric about the point $(0;0.5)$. This implies that the segment of the line above $w = 0.5$ will always be unstable to at least one of the other two strategies. Hence, the stable segment ranges between $(0 \leq w \leq w_{RC} \leq 0.5)$.

In summary, non-rewarding players (D, C) fare better than their respective rewarding type (RD, RC). Defecting types (D, RD) fare better than their respective contributing type (C, RC). Antisocial defectors (RD) fare better than cooperators (C). Finally, how rewarding cooperators (RC) fare against defectors (D) depends on whether they are able to compensate or even outweigh the costs of contribution with their reward.

Let us now determine what happens on each face and in the interior of the simplex $S_4$. The face D-C-RC: $y = 0$, corresponds to the basic model of reward fund of Sasaki & Unemi\(^1\). The population will end up in the vertex D unless $c_2(r_2 - 1) - \sigma > 0$
(Figure S1-S3). Under this condition and provided $r_2 > N$, RC becomes the global attractor, no interior fixed points exist and all interior orbits converge to RC. If $r_2 < N$, however, then the edges form a cycle of rock-scissors-paper type from C to D, D to RC and RC to C (Figure S4). In the interior of the face, a mixture equilibrium $Q_1$ of the three strategies, which is neutrally stable, appears and is surrounded by closed orbits¹ (Figure S4). Numerical investigations show that interior states of S4 will end up cycling around this equilibrium for low values of $r_2$. Otherwise, interior states will end up on the face D-RD-RC (figure S2).

On the face D-RD-RC: $z = 0$, $P_D > P_{RC}$ if $c_2(r_2 - 1) < \sigma$ and D will be the end point of the system and the evolution on the edge D-RD will go from RD to D (Figure S1). In the boundary case that $c_2(r_2 - 1) - \sigma = 0$, the D-RC edge becomes a line of fixed points (Figure S3), separated into an unstable segment ($w_{RC} < w < 1$) and a stable one ($0 \leq w \leq w_{RC}$). Interior states converge to the stable segment (see above) and the system will ultimately end up in the D vertex (Figure S3). When $c_2(r_2 - 1) > \sigma$, the edges form a rock-paper-scissors cycle from RD to D, D to RC and RC to RD (Figure S4). In order to analyze the dynamics in the interior of the face, I introduce the variable $f_D = y/(1 - w)$ representing the fraction of defectors in the PGG that are also rewarders. This yields

$$\dot{f}_D = -\frac{xy}{(1 - w)^2}(P_D - P_{RD}) = -f_D(1 - f_D)F_D(w). \tag{S13}$$

By substituting $y = f_D(1 - w)$, $z = 0$ and equation (S9) into $\dot{w} = w(P_{RC} - P)$, this yields

$$\dot{w} = -w(1 - w)[\sigma - c_2(r_2 - 1)(1 - f_D)]. \tag{S14}$$
Then, solving $\dot{y} = 0$ yields $\tilde{f}_D := 1 - \frac{\sigma}{c_2(r_2 - 1)}$, which represents the fraction of anti-social rewarders at the equilibrium. Since $F_D(w)$ has one root (see above), there is an interior fixed point $Q_2$ determined by

\[
\hat{x} = (1 - \tilde{f}_D)(1 - \hat{w}), \quad \hat{y} = \tilde{f}_D(1 - \hat{w}). \tag{[S15]}
\]

We can see that this interior equilibrium will come out of the edge D-RC (i.e. $f_0 = 0$) as the net reward exactly outweighs the net cost of contribution $\sigma$ (Figure S4). Numerical investigations show that interior states of S4 will end up cycling around this equilibrium for high values of $r_2$. Otherwise, interior states will end up on the face D-C-RC (figure S2).

On the face D-C-RD: $w = 0$, resulting in $\dot{w} = -z[c_2(r_2 - 1)y + \sigma(1 - z)] < 0$.

Therefore, cooperators C always fare worse than pure defectors D and antisocial rewarders RD as those two reap the benefit of the PGG without paying the cost. Thus, there are no interior fixed point and all interior states move away from the vertex C. For high values of $z$ interior states go towards the vertex RD, but as $z$ decreases they move toward the vertex D (Figures S1, S3 and S4).

On the face C-RD-RC: $x = 0$, resulting in $\dot{y} = y[\sigma(1 - y) + c_2(r_2 - 1)z] > 0$.

Therefore, there are no interior fixed points and all interior states converge toward the vertex RD (Figures S1, S3 and S4). Here, RD not only reap the benefits from the PGG but they also reward themselves which increases even more their advantage over C and RC.

In order to analyze the dynamics inside the simplex let us introduce a new variable $g_c = z + w$, which represents the proportion of contributors to the PGG in the population. Substituting $g_c, f_0 = y/(x + y)$ and $f_c = w/(z + w)$ into equation (S9) yields...
\[ \bar{P}_2 = \bar{g}_c [(1 - \bar{f}_c)P_c + \bar{f}_cP_{RC}] + (1 - \bar{g}_c)[(1 - \bar{f}_D)P_D + \bar{f}_D P_{RD}] \]
\[ = \bar{g}_c [(\bar{f}_c - \bar{f}_D)c_2(r_2 - 1) + c_1(r_1 - 1)] + c_2(r_2 - 1)f_D. \]  

[S16]

Therefore,
\[ \tilde{g}_c = \bar{g}_c\left[\left((1 - \bar{f}_c)P_c + \bar{f}_cP_{RC}\right) - \bar{P}_2\right] = \bar{g}_c(1 - \bar{g}_c)[c_2(r_2 - 1)(\bar{f}_c - \bar{f}_D) - \sigma]. \]  

[S17]

Solving \( \tilde{g}_c = 0 \) yields
\[ \tilde{f}_D = \tilde{f}_c - \frac{\sigma}{c_2(r_2 - 1)}. \]  

[S18]

Consequently, an interior equilibrium can exist between contributors and defectors in the PGG whenever \( \tilde{f}_c > \frac{\sigma}{c_2(r_2 - 1)} \) and provided \( c_2(r_2 - 1) > \sigma \). If \( c_2(r_2 - 1) \leq \sigma \), the population will end up in the vertex D (Figure S1 and S3).

Remember that \( F_c(z + w) \) and \( F_D(z + w) \) are symmetric in the open interval (0,1) about the vertical line \( \frac{1}{2} \), which means that their intersection point lies on this line.

Consequently, the proportion of contributors \( \bar{g}_c \) at the equilibrium is given by
\[ \bar{g}_c = z + w = x + y = \frac{1}{2}. \]  

Substituting \( z + w \) by \( \frac{1}{2} \) into \( F_c(z + w) \) and solving \( F_c(z + w) = 0 \) yields
\[ \bar{r}_2 = \frac{N2^{N-1}}{2^N - 1} < N. \]  

[S19]

We can now derive \( \hat{y} \) for example, by substituting \( z + w = \frac{1}{2}, f_D = y/(1 - z - w) \) and \( f_c = w/(z + w) \) into equation (S18), which gives
\[ \hat{y} = \hat{w} - \frac{\sigma}{2c_2(r_2 - 1)}. \]  

[S20]
Consequently, there will be interior equilibria (i.e. \(0 < \hat{y} < \frac{1}{2}\)) if \(\tilde{\nu} > \frac{\sigma}{2c_2(r_2 - 1)}\).

In summary, if (i) \(\sigma < c_2(r_2 - 1)\), (ii) \(w > \sigma/2c_2(r_2 - 1)\) and (iii) \(r_2 = N2^{N-1}/(2^N - 1)\) there will be a line of mixture equilibria of the four strategies in the interior of \(S_4\) between \(f_C = \sigma/c_2(r_2 - 1)\) and \(f_C = 1\) where \(z + w = x + y = \frac{1}{2}\) and \(y = w - \sigma/2c_2(r_2 - 1)\).

Numerical investigations show that this line is unstable and that surrounding orbits move away from the face D-C-RC and go toward the face D-RD-RC (Figure S2).

**Section 2: Different rewarding abilities.** In this second scenario I assume that defects in the PGG have their own cost \(c_3\) and multiplication factor \(r_3\) for the anti-social rewarding fund. Obviously, results from the basic model on the faces of \(S_4\) where RD and RC are not present together stay the same, i.e. \(c_2\) and \(r_2\) are simply replaced by \(c_3\) and \(r_3\) in D and RD’s payoff. The cases where the dynamics are altered by this assumption are on the edge RC-RD, the faces D-RD-RC and C-RD-RC, as well as in the interior of \(S_4\).

On the edge RD-RD: \(x + z = 0\), resulting in \(\dot{\nu} = (P_{RC} - P_{RD})w(1 - w) = w(1 - w)[c_2(r_2 - 1) - \sigma - c_3(r_3 - 1)]\). Therefore, the direction of the evolution on this edge will depend on whether RC can outweigh the net cost of the PGG with their reward as well as compensate for the defectors’ net reward. That is, if \(c_2(r_2 - 1) - \sigma < c_3(r_3 - 1)\), the direction will go from RC to RD. If \(c_2(r_2 - 1) - \sigma > c_3(r_3 - 1)\), the direction will go from RD to RC. In the boundary case that \(c_2(r_2 - 1) - \sigma = c_3(r_3 - 1)\), the edge becomes a line of fixed points. By replacing \(c_2\) by \(c_3/(r_3 - 1) + \sigma\) into \(P_D, P_{RD}, P_C\) and \(P_{RC}\), one can determine that this line will be stable against D if \(r_3(1 - w^N) - N(1 - w) > 0\) and against C if \(r_2[1 - (1 - w)N] - Nw > 0\). Those two functions are not necessarily symmetric any longer (as \(M_C(w)\) and \(M_{RD}(w)\) were), so determining the stability of the edge can only be done numerically.
On the face D-RD-RC, replacing $c_2$ and $r_2$ by $c_3$ and $r_3$, respectively, and substituting $f_D = y/(1 - w)$, $z = 0$ and equation (S9) into $\dot{w} = w(P_{RC} - \bar{P})$ yields

$$\dot{w} = w(1 - w)[c_2(r_2 - 1) - \sigma - c_3(r_3 - 1)f_D].$$ \[S21\]

Solving $\dot{w} = 0$ yields $\tilde{f}_{D_2} := \frac{c_3(r_3 - 1) - \sigma}{c_3(r_3 - 1)}$, which represents the proportion of defectors in the PGG that are also rewarders at the equilibrium. We know that $F_D(w)$ has one root, therefore there is a fixed point in the interior of the face (i.e. $0 < \tilde{f}_{D_2} < 1$) if $c_3(r_3 - 1) > c_2(r_2 - 1) - \sigma$ and $c_2(r_2 - 1) > \sigma$, which lies on the line $y = x\tilde{f}_{D_2}/(1 - \tilde{f}_{D_2})$. However, if $c_3(r_3 - 1) < c_2(r_2 - 1) - \sigma$, numerical investigations show that all interior states converge toward the vertex RC (Figure S6). In the boundary case that $c_3(r_3 - 1) = c_2(r_2 - 1) - \sigma$ however, the RD-RC edge is divided into a stable segment ($w_{RC} < w \leq 1$) and an unstable one ($0 \leq w \leq w_{RC}$). Interior states converge toward the stable one, and small perturbations will eventually lead the population to the vertex RC.

On the face C-RD-RC, $x = 0$, substituting $f_C = w/(1 - y)$ into $\dot{y} = y(P_{RD} - \bar{P})$ yields

$$\dot{y} = y(1 - y)[c_3(r_3 - 1) - c_2(r_2 - 1)f_C + \sigma].$$ \[S22\]

Solving $\dot{y} = 0$ yields $\tilde{f}_{C_2} := \frac{c_3(r_3 - 1) + \sigma}{c_3(r_3 - 1)} > 0$, which represents the proportion of cooperators in the PGG that are also rewarders at the equilibrium. Again, since $F_C(w)$ has one root, there is a fixed point $Q_2$ in the interior of the face (i.e. $\tilde{f}_{C_2} < 1$) if $c_3(r_3 - 1) < c_2(r_2 - 1) - \sigma$, which lies on the line $y = 1 - w/\tilde{f}_{C_2}$ (Figure S6). Numerical investigations show that surrounding orbits move away from this point, and go toward the edges, which form a rock-paper-scissors cycle (Figure S6). If $c_3(r_3 - 1) > c_2(r_2 - 1) - \sigma$, then all interior orbits converge toward the vertex RD. In the boundary case that $c_3(r_3 - 1) =$
$c_2(r_2 - 1) - \sigma$, the RD-RC edge becomes a line of fixed points separated into a stable ($0 \leq w \leq w_{RC}$) and an unstable segment ($w_{RC} < w \leq 1$). Interior states converge toward the stable one, and small perturbations will eventually lead the population to the vertex RD.

In the interior of S4, replacing $c_2$ and $r_2$ by $c_3$ and $r_3$ respectively in equation (S9) yields

$$\bar{P}_3 = g_c[c_2(r_2 - 1)f_c + c_1(r_1 - 1) - c_3(r_3 - 1)f_D] + c_3(r_3 - 1)f_D. \quad [S23]$$

Therefore,

$$g_{c_2} = g_c[(1 - f_c)P_c + f_c P_{RC}) - \bar{P}_3]$$

$$= g_c(1 - g_c)[c_2(r_2 - 1)f_c - \sigma - c_3(r_3 - 1)f_D]. \quad [S24]$$

$$\dot{f}_c = -f_c(1 - f_c)c_2 \left(1 - \frac{r_2 1 - (1 - g_c)^N}{N g_c} \right) \quad [S25]$$

and

$$\dot{f}_D = -f_D(1 - f_D)c_3 \left(1 - \frac{r_3 1 - g_c^N}{N 1 - g_c} \right) \quad [S26]$$

Solving $g_{c_2} = 0$ yields

$$\tilde{f}_{D_3} := \frac{c_2(r_2 - 1)f_c - \sigma}{c_3(r_3 - 1)}. \quad [S27]$$

with $0 < \tilde{f}_{D_3} < 1$ if
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\[
\frac{\sigma}{c_2(r_2 - 1)} < \tilde{f}_C < \frac{\sigma + c_3(r_3 - 1)}{c_2(r_2 - 1)}.
\]

[S28]

Solving \( \tilde{f}_C = 0 \) and \( \tilde{f}_D = 0 \) gives the conditions for rewarding individuals to be at the equilibrium with their respective non-rewarding types. Specifically, RC will be at the equilibrium with C if

\[
H_{RC}(z + w) := \frac{(z + w)N}{1 - [1 - (z + w)]N} - r_2 = 0
\]

[S29]

and RD will be at the equilibrium with D if

\[
H_{RD}(z + w) := \frac{[1 - (z + w)]N}{1 - [z + w]^N} - r_3 = 0.
\]

[S30]

If \( r_2 = r_3, H_{RC}[1/2 - (z + w)] = -H_{RD}[1/2 + (z + w)]. \) Therefore, \( H_{RC} \) has a center of symmetry with respect to \( H_{RD} \) about the point \( z + w = \frac{1}{2} \). Otherwise, the equilibrium point \( \tilde{g}_C = z + w \) will be above \( \frac{1}{2} \) if \( r_2 > r_3 \) and below \( \frac{1}{2} \) if \( r_2 < r_3 \). Numerical investigation show that interior states end up cycling either on the face D-RD-RC when \( c_2(r_2 - 1) - \sigma < c_3(r_3 - 1) \), or on the face RD-C-RC otherwise (Figure S5).

Section 3: Exclusion of second-order free-riders from the rewarding fund. In this third scenario, I investigate the setting where rewarding players can fully or partially exclude their respective non-rewarding type. This solution, proposed by Sasaki & Unemi, has proven effective in the absence of anti-social rewarding when rewarding players face a social dilemma (i.e. \( r_2 < N \)) if rewarders can exclude a sufficient proportion of the non-rewarding players' reward. I introduce the new parameter \( \alpha \) \((0 \leq \alpha \leq 1)\) which represent the proportion of the reward that rewarding players can exclude from their
corresponding non-rewarding type. Consequently, cooperators $C$ have now an expected reward given by

$$P_{C_2} = (1 - \alpha)r_2c_2 \left(1 - \frac{1 - (x + y)^N}{N(z + w)}\right)\left(\frac{w}{z + w}\right),$$ \[S31\]

and defectors $D$ have an expected reward given by

$$P_{D_2} = (1 - \alpha)r_2c_2 \left(1 - \frac{1 - (z + w)^N}{N(x + y)}\right)\left(\frac{y}{x + y}\right).$$ \[S32\]

Cases that are altered by these new assumptions are when rewarding players and their corresponding non-rewarding types find themselves together: the edges $D\text{-}RD$ and $C\text{-}RC$, the faces $D\text{-}C\text{-}RC$, $D\text{-}RD\text{-}C$, $D\text{-}RD\text{-}RC$ and $C\text{-}RC\text{-}RD$, as well as in the interior of $S_4$.

On the edges $D\text{-}RD$ and $C\text{-}RC$: $z + w = 0$ and $x + y = 0$ resulting respectively in

$$\dot{y} = (P_{RD} - P_D)y(1 - y) = y(1 - y)c_2 \left[\frac{r_2}{N} - 1 + \alpha(N - 1)y \frac{r_2}{N}\right]$$ \[S33\]

and

$$\dot{w} = (P_{RC} - P_C)w(1 - w) = w(1 - w)c_2 \left[\frac{r_2}{N} - 1 + \alpha(N - 1)w \frac{r_2}{N}\right].$$ \[S34\]

Since $r_2 < N$, there will be an unstable fixed point $\frac{1}{\alpha(N-1)} \left(\frac{N}{r_2} - 1\right)$ and $\frac{1}{\alpha(N-1)} \left(\frac{N}{r_2} - 1\right)$ on each edge, respectively. Those fixed points will exist on their respective edges (i.e. $0 < \frac{1}{\alpha(N-1)} \left(\frac{N}{r_2} - 1\right)$ and $0 < \frac{1}{\alpha(N-1)} \left(\frac{N}{r_2} - 1\right)$ if $\alpha > (N - r_2)/(r_2(N - 1))$. Otherwise, $D$ and $C$ will take over their respective rewarding type.
The dynamics on the face D-C-RC correspond to the basic model in Sasaki & Unemi\(^3\). Defectors will dominate unless \(c_2(r_2 - 1) > \sigma\). For small \(\alpha\) values, the edges form a rock-paper-scissors cycle, and an unstable interior point exists\(^4\), with surrounding orbits moving away from it (Fig. 1 in the main text). As seen above, as \(\alpha\) crosses the threshold given by \((N - r_2)/[r_2(N - 1)]\), a fixed point enters the edge C-RC and drives the population toward the vertex RC (Figure 1 in the main text).

On the face D-C-RD: \(w = 0\), and \(P_D - P_C > 0\). Therefore, C-players will always fare worse than D-players. Consequently, there are no interior fixed points and all interior states move away from the vertex C. Whether the system will end up in the vertex D or RD will depend on \(\alpha\). The outcome will be D if \(\alpha \leq (N - r_2)/[r_2(N - 1)]\), otherwise the interior states will end up either on the vertex D or RD, depending on the initial conditions.

On the face D-RD-RC: \(z = 0\). The system will eventually end up in the vertex D if \(c_2(r_2 - 1) - \sigma \leq 0\) and \(\alpha \leq (N - r_2)/[r_2(N - 1)]\). If \(c_2(r_2 - 1) - \sigma \leq 0\) and \(\alpha > (N - r_2)/[r_2(N - 1)]\), the edge D-RD has an unstable fixed point \(\gamma_{RD}^{\infty} = \frac{1}{\alpha(N-1)} \left(\frac{N}{r_2} - 1\right)\) and D or RD will be the outcome depending on the initial conditions. If \(c_2(r_2 - 1) - \sigma > 0\) and \(\alpha \leq (N - r_2)/[r_2(N - 1)]\), then all edges form a rock-paper-scissors cycle, from D to RC, RC to RD and RD to D, and numerical investigations show that there is an interior fixed point, with surrounding orbits moving away from it (Figure S7). Finally, when \(c_2(r_2 - 1) - \sigma > 0\) and \(\alpha > (N - r_2)/[r_2(N - 1)]\), the D-RD edge has an unstable fixed point and small perturbations will eventually lead interior states toward the vertex RD (Figure 1 in the main text).
On the face C-RD-RC: $x = 0$, substituting $f_c = w/(1 - y)$ into $\dot{y} = y(P_{RD} - \bar{P})$ results in

$$\dot{y} = yr_2c_2(1 - f_c) \left[ \frac{\sigma(1 - y)}{r_2c_2(1 - f_c)} + \alpha f_c \left( 1 - \frac{1 - y}{N} - y \right) + \left( 1 - \frac{1}{r_2} \right)(1 - y) \right] > 0. \quad [S35]$$

Therefore, there are no interior points and the vertex RD will be the global attractor.

In the interior of $S_4$, substituting $g_c = z + w$ and equation (S31) into $\dot{f}_c = f_c [P_{RC} - (f_cP_{RC} + (1 - f_c)P_c)]$ and $\dot{f}_D = f_D [P_{RD} - (f_DP_{RD} + (1 - f_D)P_D)]$ yields respectively

$$\dot{f}_c = f_c(1 - f_c)c_2r_2 \left[ (1 - \alpha f_c) \frac{1 - (1 - g_c)^N}{g_c N} + \alpha f_c - \frac{1}{r_2} \right]. \quad [S36]$$

and

$$\dot{f}_D = f_D(1 - f_D)c_2r_2 \left[ (1 - \alpha f_D) \frac{1 - g_c^N}{(1 - g_c)N} + \alpha f_D - \frac{1}{r_2} \right]. \quad [S37]$$

Solving $\dot{f}_c = 0$ and $\dot{f}_D = 0$ yields respectively

$$\hat{f}_c = \frac{Ng_c - r_2[1 - (1 - g_c)^N]}{\alpha r_2[(1 - g_c)^N + Ng_c - 1]} \quad [S38]$$

and

$$\hat{f}_D = \frac{N(1 - g_c) - r_2(1 - g_c^N)}{\alpha r_2[g_c^N + N(1 - g_c) - 1]} \quad [S39]$$
Equations (S38) and (S39) are symmetric about the vertical line $g_C = 0.5$ in the open interval $(0,1)$, which implies that their intersection point will be on this line. Substituting $g_C = 0.5$ into equation (S38) and solving $\tilde{f}_C > 0$ yields

$$\tilde{r}_2 > \frac{2^N N}{2^{N+1} - 2} \quad [S40]$$

which is the condition such that the sign of $\tilde{f}_C$ will always be the opposite of the sign of $\tilde{f}_D$, or in other words, when there cannot be an interior equilibrium (because $\tilde{f}_D < 0$ if $\tilde{f}_C > 0$ or $\tilde{f}_C < 0$ if $\tilde{f}_D > 0$). In addition, substituting $g_C = 0.5$ into equation (S38) and solving $\tilde{f}_C < 1$ yields

$$\alpha < \frac{2^N N - 2r_2(2^N - 1)}{r_2[2^N N - 2(2^N - 1)]} \quad [S41]$$

which is the condition such that either $\tilde{f}_C$ or $\tilde{f}_D$ is greater than 1, and hence no interior points could exist. Finally, the condition for contributors to be at the equilibrium with non-contributors is mathematically too complicated to be insightful. Numerical investigations show that there is at most one unstable interior fixed point with $\tilde{g}_C \geq 0.5$, and that surrounding orbits move towards the face D-RD-RC for intermediate values of $r_2$.

Section 4: Reward fund investment as fraction of the per capita PGG share. In this final condition, I make the assumption that individuals can only invest a fraction $\beta$, with $0 \leq \beta \leq 1$, of their PGG share into the reward fund. Hence they will only get a fraction $(1 - \beta)$ of the public good. With this new condition we need to redefine all payoffs.
The expected payoffs for the PGG stay the same for non-rewarding players. Rewarding players now invest a fraction of their PGG share:

\[
P_{D1} = \frac{r_1 c_1}{N} (N - 1)(z + w) \tag{S42}
\]

\[
P_{RD1} = (1 - \beta) \frac{r_1 c_1}{N} (N - 1)(z + w) \tag{S43}
\]

\[
P_{C1} = \frac{r_1 c_1}{N} [(N - 1)(z + w) + 1] - c_1 \tag{S44}
\]

\[
P_{RC1} = (1 - \beta) \frac{r_1 c_1}{N} [(N - 1)(z + w) + 1] - c_1 \tag{S45}
\]

Note that the fraction of the PGG that RC players invest includes their own contribution. For the rewarding stage, a C-player in a group with \(N_C\) contributors (C and RC, focal included) and \(N_R\) rewarding cooperators will receive a reward of \(\beta N_C (r_1 c_1 / N) r_2 N_R / N_C (0 \leq N_R \leq N_C - 1)\). Therefore, the expected reward for a C-player is given by

\[
P_{C2} = r_2 \beta \frac{r_1 c_1}{N} (N - 1)w. \tag{S46}
\]

An RC-player in a group with \(N_C\) contributors (C and RC, focal included) and \(N_R\) other rewarding players (focal excluded) will receive a reward of \(\beta N_C (r_1 c_1 / N) r_2 (N_R + 1) / N_C\). Therefore, the expected reward for an RC-player is given by

\[
P_{RC2} = r_2 \beta \frac{r_1 c_1}{N} [(N - 1)w + 1]. \tag{S47}
\]
For defectors, whether they are going to receive a share of the anti-social reward fund will depend on the PGG and therefore the presence of contributors. A D-player in a group with \( N_D \) non-contributors (i.e. D and RD, focal included) and \( N_{RD} \) other rewarding defectors, and therefore \( N - N_D \) contributors, will receive a reward of \( r_2 \beta (N - N_D)(r_1 c_1 / N) N_{RD} / N_D \). Therefore, the expected reward for a D-player in a group with \( N_D \) non-contributors is

\[
P_{D2}(N_D) = \sum_{N_{RD}=0}^{N_D-1} \left( \frac{N_D - 1}{N_{RD}} \right)^{N_{RD}} \left( \frac{y}{x + y} \right)^{N_{RD}} \left( \frac{x}{x + y} \right)^{N_D - N_{RD} - 1} r_2 \beta \frac{r_1 c_1}{N} (N - N_D) \frac{N_{RD}}{N_D}
\]

\[
= r_2 \beta \frac{r_1 c_1}{N} (N - N_D) \left( \frac{x}{x + y} \right)
\]

where

\[
\sum_{N_{RD}=0}^{N_D-1} \left( \frac{N_D - 1}{N_{RD}} \right)^{N_{RD}} \left( \frac{y}{x + y} \right)^{N_{RD}} \left( \frac{x}{x + y} \right)^{N_D - N_{RD} - 1}
\]

is the probability of that \( N_{RD} \) of the other non-contributors are rewarders. Hence, the expected reward for a D-player is given by

\[
P_{D2} = \sum_{N_{D}=1}^{N} \left( \frac{N - 1}{N_{D} - 1} \right) (x + y)^{N_{D} - 1} (z + w)^{N - N_D} P_{D2}(N_D)
\]

\[
= r_2 \beta \frac{r_1 c_1}{N} \left[ 1 - \frac{1 - (z + w)^N}{x + y} + (N - 1)(z + w) \right] \left( \frac{y}{x + y} \right).
\]

The expected reward that RD-players will get from others, \( P_{RD2} \), is equal to \( P_{D2} \) and, in addition, they will get their own contribution to the anti-social reward fund.
Thus, the expected return for RD-players $P_{RD3}$ coming from their own contribution to the reward fund in a group with $N_D$ non-contributors (focal included) is

$$P_{RD3}(N_D) = \sum_{N_D=1}^{N} \left( \frac{N - 1}{N_D - 1} \right) (x + y)^{N-D-1}(z + w)^{N-N_D} r_2 \frac{r_1 c_1}{N} (N - N_D) \frac{1}{N_D}$$

$$= -r_2 \beta \frac{r_1 c_1}{N} \left( 1 - \frac{1 - (z + w)^N}{x + y} \right).$$  \[S50\]

We can now determine that $P_D = P_{D1} + P_{D2}$, $P_{RD} = P_{RD1} + P_{RD2} + P_{RD3}$, $P_C = P_{C1} + P_{C2}$ and $P_{RC} = P_{RC1} + P_{RC2}$. The average payoff in the population is now

$$\bar{P}_C = g_c c_1 r_1 (r_2 - 1) \left[ \frac{r_1 - 1}{\beta r_1 (r_2 - 1)} + (f_c - f_D) \left( g_c + \frac{(1 - g_c)}{N} \right) + f_D \right].$$  \[S51\]

where $g_c = z + w$, $f_c = w/ (z + w)$ and $f_D = y/ (x + y)$.

Results on the edge C-D stay the same, as there are no rewarding players. However, since rewards depend on the PGG, the edge D-RD is now a line of fixed points as both have a zero payoff (Figure S5). This line will always be stable against C but will be unstable with respect to RC if $\beta > (N - r_1)/ r_1 (r_2 - 1)$.

On the edge D-RC: $y + z = 0$ resulting in

$$\dot{w} = (P_{RC} - P_D) w (1 - w) = w (1 - w) \left[ \frac{c_1 r_1}{N} \beta (r_2 - 1) \left((N - 1)w + 1 \right) - \sigma \right].$$  \[S52\]

Therefore, there will be an unstable equilibrium given by

$$\bar{w}_{RC} = \frac{N - r_1 \beta (r_2 - 1) + 1}{r_1 (N - 1) \beta (r_2 - 1)}.$$  \[S53\]
If
$$\frac{N - r_1}{r_1 N (r_2 - 1)} < \beta < \frac{N - r_1}{r_1 (r_2 - 1)}.$$ \[S54\]

If \(\beta \leq \frac{(N - r_1)}{r_1 N (r_2 - 1)}\), the direction of the evolution goes from RC to D. If \(\beta \geq \frac{(N - r_1)}{r_1 (r_2 - 1)}\), the direction of the evolution goes from D to RC. Note that \(\frac{(N - r_1)}{r_1 (r_2 - 1)}\) will be in the open interval \((0,1)\) if \(r_1 r_2 \geq N\).

On the edge C-RC: \(x + y = 0\) resulting in

$$\dot{w} = (P_{RC} - P_D)w(1 - w) = -w(1 - w)c_1 r_1 \beta \left(1 - \frac{r_2}{N}\right) < 0.$$ \[S55\]

Therefore, the direction of the evolution on this edge will go from RC to C.

On the edge RD-C: \(x + w = 0\) resulting in

$$\dot{z} = (P_C - P_{RD})z(1 - z) = -z(1 - z) \left[\sigma + \frac{r_1 c_1}{N}(N - 1)\beta (r_2 - 1)z\right] < 0.$$ \[S56\]

Therefore, the direction of the evolution will go from C to RD.

On the edge RD-RC: \(x + z = 0\) resulting in

$$\dot{w} = (P_{RC} - P_{RD})w(1 - w) = w(1 - w)\left[\frac{r_1 c_1}{N}\beta (r_2 - 1) - \sigma\right].$$ \[S57\]

Therefore, the direction of the evolution will go from RC to RD if \(\beta < \frac{(N - r_1)}{r_1 (r_2 - 1)}\), and from RD to RC if \(\beta > \frac{(N - r_1)}{r_1 (r_2 - 1)}\). In the boundary case that \(\beta = \frac{(N - r_1)}{r_1 (r_2 - 1)}\) the whole edge will be a line of fixed points, which will be stable against D and C if

\(r_2 (w - w^N) - w[N(1 - w) + w - 1] > 0\) and \(w > (r_2 - 1)/(N - 1)\), respectively.
On the face D-C-RC: $y = 0$, substituting $g_c = 1 - x$ and $f_c = w/(1 - x)$ into

\[\frac{df_c}{dt} = f_c[P_{RC} - (f_c P_{RC} + (1 - f_c)P_C)] \text{ results in}\]

\[\hat{f}_c = -f_c(1 - f_c)\beta \frac{r_1 c_1}{N} [1 + g_c(N - 1) - r_2]. \tag{S58}\]

Solving $\hat{f}_c = 0$ yields

\[\hat{g}_c = \frac{r_2 - 1}{N - 1} \tag{S59}\]

which represents the proportion of contributors to the PGG at the equilibrium. In addition,

\[\hat{g}_c = g_c\{[f_c P_{RC} + (1 - f_c)P_C] - [g_c f_c P_{RC} + (1 - f_c)P_C] + (1 - g_c)P_D\}
\]

\[= g_c(1 - g_c) r_1 c_1 \beta (r_2 - 1)(g_c(N - 1) + 1) + 1 - \frac{N}{r_1}. \tag{S60}\]

Substituting equation (S59) into equation (S60) and solving $\hat{g}_c = 0$ yields

\[\hat{f}_c = \frac{N - r_1}{\beta r_1 r_2 (r_2 - 1)}. \tag{S61}\]

Therefore, D will be the global attractor for small values of $\beta$ (Figures S8). Then, if $\beta$ equals a threshold given by $(N - r_1)/r_1 r_2 (r_2 - 1)$, an unstable fixed point appears on the edge D-RC, but D will still be the outcome. If $\beta > (N - r_1)/r_1 r_2 (r_2 - 1)$, an unstable interior equilibrium will come out of the edge D-RC (Figure S11).

On the face D-C-RD: $w = 0$, and substituting $\hat{f}_D = y/(1 - z)$ into

\[\hat{z} = z(P_C - \bar{P}) \text{ yields}\]

\[\hat{z} = -z(1 - z) \left[\sigma + \frac{r_1 c_1}{N} \beta f_D(N - 1)(r_2 - 1)z\right] < 0. \tag{S62}\]
Therefore, there are no interior fixed points and all interior states move away from the vertex C, and go toward the edge D-RD which is a line of fixed points (Figure S8 and S11).

On the face D-RD-RC: \( z = 0 \), resulting in

\[
\dot{w} = w(1 - w)\left[\frac{r_1 c_1}{N} \beta (r_2 - 1)(1 + (1 - f_D)(N - 1)w) - \sigma\right].
\]  \[\text{[S63]}\]

Solving \( \dot{w} = 0 \) yields

\[
\bar{f}_D = 1 + \frac{r_1[1 + \beta(r_2 - 1)] - N}{\beta(r_2 - 1)r_1(N - 1)w}.
\]  \[\text{[S64]}\]

\( \bar{f}_D \) will be in the open interval \((0,1)\) if

\[
\frac{N - r_1}{r_1(r_2 - 1)[1 + (N - 1)w]} < \beta < \frac{N - r_1}{r_1(r_2 - 1)}.
\]  \[\text{[S65]}\]

In addition,

\[
\dot{f}_D = \frac{xy}{(1 - w)^2} (p_{RD} - p_D)
= (1 - f_D)\bar{f}_D \frac{\beta}{1 - w} \frac{c_1 r_1}{N} [(r_2 + 1 - N + (N - 1)w)w - r_2 w^N].
\]  \[\text{[S66]}\]

Solving \( \dot{f}_D = 0 \) yields

\[
F_{D_2}(w) := r_2 - \frac{w(1 - w)(N - 1)}{w - w^N} = 0.
\]  \[\text{[S67]}\]
The function $F_{D_2}(w)$ has a unique root $\hat{w}$ in the open interval $(0,1)$ if, and only if, $r_2 < N - 1$, since $F_{D_2}(w)$ is monotonic, $F_{D_2}(0) = r_2 - (N - 1) < 0$, and $F_{D_2}(1) = r_2 - 1 > 0$. This root depends on the value of $r_2$, i.e. $r_{2T}$, which can only be determined numerically. Numerical investigations show that this interior equilibrium lies on the line $y = \hat{f}_D(1 - w)$ and is unstable with respect to the fourth strategy $C$. Surrounding orbits move away from it and end up on the edge D-RD. However, if $\beta \geq (N - r_1)/(r_1(r_2 - 1))$, all interior states converge toward the vertex RC (Figure S11).

On the face C-RD-RC: $x = 0$, and substituting $f_C = w/(1 - y)$ into $\dot{y} = y(P_{RD} - \bar{P})$ and solving $\dot{y} = 0$ yields

$$\hat{f}_C = 1 - \frac{r_1[1 + \beta(r_2 - 1)] - N}{\beta(r_2 - 1)r_1[N - y(N - 1)]} \quad \text{[S68]}$$

Therefore, there can be an interior equilibrium (i.e. $0 < \hat{f}_C < 1$) if $\beta > (N - r_1)/r_1(r_2 - 1)$. In addition,

$$\dot{f}_C = \frac{zw}{(1 - y)^2}(P_{RC} - P_C) = (1 - f_C)\frac{c_1r_1}{N}\beta[r_2 - N + y(N - 1)]. \quad \text{[S69]}$$

Solving $\dot{f}_C = 0$ yields the frequency of RD at the equilibrium given by $\dot{y} = (N - r_2)/(N - 1)$, which will be in the open interval $(0,1)$ if $r_2 < N$ (Figure S11). If $\beta \leq (N - r_1)/r_1(r_2 - 1)$, then all interior states converge toward the vertex RD (Figure S8).

We can now analyze the dynamics in the interior of $S_1$. Substituting $f_C = w/(z + w)$, $f_D = y/(x + y)$ and $g_C = z + w$ into $\dot{f}_C = f_C(P_{RC} - (f_C P_{RC} + (1 - f_C)P_C))$, $\dot{f}_D = f_D(P_{RD} - (f_D P_{RD} + (1 - f_D)P_D))$ and $\dot{g}_C = g_C[(f_C P_{RC} + (1 - f_C)P_C) - \bar{P}_4]$ yields respectively

25
\[ \dot{f}_c = \frac{zw}{(z + w)^2} (p_{kc} - p_c) = f_c (1 - f_c) \beta \frac{c_1 r_1}{N} [r_2 - 1 - g_c(N - 1)], \]  
\text{[S70]} \]

\[ \dot{f}_D = \frac{xy}{(x + y)^2} (p_{RD} - p_D) = f_D (1 - f_D) \beta \frac{c_1 r_1}{N} \left[ r_2 \frac{g_c - g_c^N}{1 - g_c} - g_c(N - 1) \right] \]  
\text{[S71]} \]

and

\[ g_c = g_c(1 - g_c) \frac{r_1 c_1}{N} \left[ \beta (r_2 - 1) (f_c + g_c(N - 1)(f_c - f_D)) + 1 - \frac{N}{r_1} \right]. \]  
\text{[S72]} \]

Solving \( \dot{f}_c = 0 \) and \( \dot{f}_D = 0 \) yields respectively

\[ g_c = \frac{r_2 - 1}{N - 1} \]  
\text{[S73]} \]

which is the proportion of contributors to the PGG (i.e. \( z + w \)) where they would be at the equilibrium with each other, and

\[ g_c(N - 1) \frac{1 - g_c}{g_c - g_c^N} - r_2 = 0, \]  
\text{[S74]} \]

which can only be solved numerically. Then, substituting \( g_c = (r_2 - 1)/(N - 1) \) into equation \( \text{[S72]} \) and solving \( g_c = 0 \) yields

\[ \bar{f}_D = \frac{r_1 [1 + \beta f_c(r_2 - 1)r_2] - N}{\beta r_1 (r_2 - 1)^2} \]  
\text{[S75]} \]
which is the proportion of defectors that are also rewarders such that contributors (i.e. \( z + w \)) would be at the equilibrium with non-contributors (i.e. \( x + y \)). From equation (S75) we can determine that \( 0 < f_D < 1 \) if

\[
\frac{N - r_1}{\beta r_1 (r_2 - 1) r_2} < f_C < \frac{N + r_1 [\beta (r_2 - 1)^2 - 1]}{\beta r_1 (r_2 - 1) r_2}
\]

[S76]

where \( f_C \) is the proportion of contributors to the PGG that are also rewarders required for RD to be at the equilibrium with D.

In summary, if the conditions in equations (S73), (S74) and (S75) are met, there will be a line of equilibria where \( f_C \) is within the range given by equation (S76).

Section 5: Numerical investigation of the dynamics in the interior of S4.

Here, I describe the method used to analyze the dynamics in the interior of S4 for each variant of the reward fund, and for a large range of parameter values. Numerical simulations of the replicator dynamics were used to compute long-term average frequencies of each strategy. Specifically, for each parameter combination, simulations starting from a large set of initial conditions were run for 1000 time-steps. Long-term frequencies were then computed over these 1000 steps. The set of initial conditions comprised initial frequencies ranging from 0.025 to \( 1 - 0.025 \cdot 3 \), increasing by intervals of 0.025, resulting in a total of 8473 initial conditions, all in the interior of S4. The average frequency of each strategy was then computed over these 8473 runs. Note that these runs are numerical simulations of the deterministic model, hence no replicates are needed. The variance in cooperation frequency was computed over all runs in order to
determine whether the population would end-up in a stable state. Figures S2, S5, S7, S9 and S14 to S19 show these long-term averages for each variant of the reward fund.

Section 6: Agent-based simulations with $\beta$ as an evolvable cultural trait.

Here, I consider a Wright-Fisher process in a finitely large and well-mixed population with $M = 10^3$ individuals. Population size is constant. Generations are discrete and non-overlapping. In each generation, $k$ groups of size $N$ are randomly formed (with $k = M/N$) for a single interaction, consisting in a PGG with reward funds. Therefore, each individual interacts only once during her lifetime. At the end of each generation, individuals reproduce in proportion to their payoff, and then die. More precisely, a baseline payoff of $1 + c_1$ is added to everybody’s payoff. Each individual’s payoff is divided by the sum of all payoffs. Then, individuals of the current generation are selected with replacement in proportion to their payoff to be the parent of a new offspring. This process is repeated until the offspring population reaches $n$.

Individuals are characterized by two cultural traits that are transmitted vertically from parent to offspring with errors during transmission (i.e. mutations). The first trait, namely $\gamma$, determines whether the individual will contribute to the PGG (i.e. either C or D for ‘cooperate’ or ‘defect’ respectively). A mutation on this trait replaces C or D by D or C, respectively. The second trait, namely $\beta$, is a continuous number varying between 0 and 1 and which determines the proportion of the received share from the PGG that will be invested in the corresponding reward fund (i.e. pro-social reward fund if the individual contribution trait is C or anti-social if the individual contribution trait is D). Since the per capita share of the public good is $B = r_1 c_1 / N N_C$ (where $N_C$ includes the focal player if she is a cooperator), then a rewarding player will get $(1 - \beta)B$ from the PGG and will invest $\beta B$ into rewarding other players of her own type (i.e. cooperator or defector). A mutation
on this trait changes its value according to a normally distributed variable centered around the current trait value and with variance \( \text{var} = 0.1 \). Results are shown in Figures S12 and S13.

References

Figure S1. Anti-social rewarding in the public goods game with reward funds. Defection prevails when contributors cannot compensate for the cost of contribution with their reward fund, i.e. \( c_2(r_2 - 1) < \sigma \). All initial states on the faces (a) as well as in the interior of \( S_4 \) (b) will end up in the vertex D. Parameters: \( N = 5 \), \( c_1 = 1 \), \( r_1 = 3 \), \( c_2 = 1 \), \( r_2 = 1.2 \).
Figure S2. Long-term average frequencies for the basic model. Cooperation frequency corresponds to the top of the blue bar (i.e. C + RC). The error bars show the standard deviation in cooperation frequency computed over all initial conditions (for each parameter combination). Here, \( c_2(r_2 - 1) > \sigma \) when \( r_2 > 1.4 \). Parameters: \( N = 5, c_1 = 1, r_1 = 3, c_2 = 1 \).
Figure S3. Anti-social rewarding in the public goods game with reward funds. In the case where contributors can exactly compensate for the cost of contribution with their reward fund (i.e. $c_2(r_2 - 1) = \sigma$) and hence have similar fitness to defectors. Hence, the edge D-RC is a line of fixed points, separated into a stable and unstable mixture segment. However, random perturbations such as mutations will eventually lead interior states on the faces as well as in the interior of S4 towards the vertex D. Parameters: $N = 5$, $c_1 = 1$, $r_1 = 3$, $c_2 = 1$, $r_2 = 1.4$. 
Figure S4. Anti-social rewarding in the public goods game with reward funds. In the case where contributors can outweigh the cost of contribution with their reward fund (i.e. $c_2(r_2 - 1) > \sigma$), the latter will have greater fitness than defectors but will still be vulnerable to second-order free-riding. Hence, non-rewarding types will fare better than rewarding ones, leading interior states of $S_4$ toward the face D-C-RC. On this face the three strategies will end up in a rock-paper-scissors cycle, around an unstable fixed point $Q_1$. Parameters: $N = 5$, $c_1 = 1$, $r_1 = 3$, $c_2 = 1$, $r_2 = 1.6$. 
Figure S5. Long-term average frequencies for the variant with different rewarding abilities for contributors and non-contributors. Cooperation frequency corresponds to the top of the blue bar (i.e. C + RC). The error bars show the standard deviation in cooperation frequency computed over all initial conditions (for each parameter combination). Here, $c_2(r_2 - 1) > \sigma$ when $r_2 > 1.4$. On each panel, $c_2(r_2 - 1) - \sigma \geq c_3(r_3 - 1)$ above the vertical bold line. Parameters: $N = 5, c_1 = 1, r_1 = 3, c_2 = c_3 = 1$. 
Figure S6. Different rewarding efficacies for contributors and non-contributors. Rewarding cooperators are able to produce more rewards and can even outweigh the anti-social reward fund, i.e. $c_3(r_3 - 1) < c_2(r_2 - 1) - \sigma$. However, second-order free-riding still occurs and leads the dynamics to a rock-paper-scissors cycle from D to RC, RC to C and C to D. Parameters: $N = 5$, $c_1 = 1$, $r_1 = 3$, $c_2 = 1$, $r_2 = 1.6$, $c_3 = 1$, $r_3 = 1.1$. 
Figure S7. Long-term average frequencies for the variant with exclusion. Cooperation frequency corresponds to the top of the blue bar (i.e. $C + RC$). The error bars show the standard deviation in cooperation frequency computed over all initial conditions (for each parameter combination). Here, $c_2(r_2 - 1) > \sigma$ when $r_2 > 1.4$. Parameters: $N = 5$, $c_1 = 1$, $r_1 = 3$, $c_2 = 1$. 
Figure S8. Dynamics in the public goods game with reward investment as a fraction $\beta$ of the produced good. Defection prevails when rewarding players can only invest a small fraction of their PGG share, as any initial condition finishes on the edge D-RD. Parameters: $N = 5$, $c_1 = 1$, $r_1 = 3$, $c_2 = 1$, $r_2 = 3$, $\beta = 0.05$. 
Figure S9. Long-term average frequencies for the variant with rewards dependent on the PGG’s production. Cooperation frequency corresponds to the top of the blue bar (i.e. C + RC). The error bars show the standard deviation in cooperation frequency computed over all initial conditions (for each parameter combination). On each panel, $\beta > (N - r_1) / r_1 r_2(r_2 - 1)$ above the vertical bold line. Parameters: $N = 5$, $c_1 = 1$, $r_1 = 3$. 
Figure S10. Contour plot of $\beta_T$ in function of the multiplication factors for the PGG ($r_1$) and reward fund ($r_2$). If reward investments can only be a fraction $\beta$ of the public good produced, a certain threshold $\beta_T = (N - r_1) / [r_1(r_2 - 1)]$ is required for pro-social rewarders to outcompete anti-social rewarders. Warmer (colder) colors indicate higher (lower) values of $\beta_T$. White areas indicate parameter values where $\beta_T$ is above 1. Parameters: $N = 5$. 
Figure S11. Dynamics in the public goods game with reward investment as a fraction $\beta$ of the produced good. When rewarders can invest a large proportion of their PGG share, the population ends up in a stable mixture $Q_2$ of cooperators, anti-social and pro-social rewarders. In the absence of contributors (i.e. C and RC), non-contributors (i.e. D and RD) produce no good that anti-social rewarders could invest in their reward fund, resulting in D and RD having similar payoffs. Hence, the edge D-RD is a line of fixed points. The line is unstable as RC mutants will have higher fitness, leading the dynamics towards the vertex RC. Parameters: $N = 5$, $c_1 = 1$, $r_1 = 3$, $c_2 = 1$, $r_2 = 3$, $\beta = 0.5$. 
Figure S12. Co-evolution of contribution to the public goods game and reward fund. Typical agent-based simulation run of the cultural evolution model, where both the contribution to the PGG and fraction of the per capita PGG share that individuals are willing to invest in their corresponding reward fund are evolvable traits ($\gamma$ and $\beta$, respectively). Starting with an initial population of non-rewarding defectors (i.e. $[\gamma, \beta] = [0, 0]$), $\beta$ will have no effect on the individuals’ fitness, since no PGG is created in the first place and nothing could be invested. Hence, random neutral mutations will drive the trait to the critical threshold $\beta_T = (N - r_1)/[r_1(r_2 - 1)]$, allowing contributors to invade. Despite not being able to completely replace non-contributors, contributors can prevent defectors to dominate again. Pro-social rewarders are still vulnerable to second-order free-riding as whenever the whole population contributes to the PGG (i.e. contribution trait close to 1), there is selection for lower $\beta$ values. Parameter values: $n = 1000$, $N = 5$, $c_1 = 1$, $r_1 = 3$, $c_2 = 1$, $r_2 = 4$, $\mu = 0.001$, $\text{var} = 0.1$. 
Figure S13. Cooperation frequency in agent-based simulations where $\beta$ is an evolvable trait. Blue bars represent long-term cooperation frequencies (± SD). Red triangles represent the population average $\beta$ values (± SD). Average cooperation frequency was computed over 10 replicates, which lasted 20'000 generations. Parameter values: $n = 1000, N = 5, c_1 = 1, r_1 = 3, c_2 = 1, \mu = 0.001, \text{var} = 0.1$. 
Figure S14. Long-term average frequencies for the variant with different rewarding abilities for contributors and non-contributors and exclusion. Cooperation frequency corresponds to the top of the blue bar (i.e. C + RC). The error bars show the standard deviation in cooperation frequency computed over all initial conditions (for each parameter combination). Here, \( c_2(r_2 - 1) > \sigma \) when \( r_2 > 1.4 \). On each panel, \( c_2(r_2 - 1) - \sigma \geq c_3(r_3 - 1) \) above the vertical bold line. Parameters: \( N = 5, c_1 = 1, r_1 = 3, c_2 = c_3 = 1, \alpha = 0.1. \)
Figure S15. Long-term average frequencies for the variant with different rewarding abilities for contributors and non-contributors and exclusion. Cooperation frequency corresponds to the top of the blue bar (i.e. C + RC). The error bars show the standard deviation in cooperation frequency computed over all initial conditions (for each parameter combination). Here, \( c_2(r_2 - 1) > \sigma \) when \( r_2 > 1.4 \). On each panel, \( c_2(r_2 - 1) - \sigma \geq c_3(r_3 - 1) \) above the vertical bold line. Parameters: \( N = 5, c_1 = 1, r_1 = 3, c_2 = c_3 = 1, \alpha = 0.5 \).
Figure S16. Long-term average frequencies for the variant with different rewarding abilities for contributors and non-contributors and exclusion. Cooperation frequency corresponds to the top of the blue bar (i.e. C + RC). The error bars show the standard deviation in cooperation frequency computed over all initial conditions (for each parameter combination). Here, \( c_2(r_2 - 1) \geq \sigma \) when \( r_2 > 1.4 \). On each panel, \( c_2(r_2 - 1) - \sigma \geq c_3(r_3 - 1) \) above the vertical bold line. Parameters: \( N = 5, c_1 = 1, r_1 = 3, c_2 = c_3 = 1, \alpha = 0.8 \).
Figure S17. Long-term average frequencies for the variant with rewards dependent on the PGG’s production and exclusion. Cooperation frequency corresponds to the top of the blue bar (i.e. C + RC). The error bars show the standard deviation in cooperation frequency computed over all initial conditions (for each parameter combination). On each panel, $\beta > (N - r_1)/r_1 r_2 (r_2 - 1)$ above the vertical bold line. Parameters: $N = 5$, $c_1 = 1$, $r_1 = 3$, $\alpha = 0.1$. 
Figure S18. Long-term average frequencies for the variant with rewards dependent on the PGG’s production and exclusion. Cooperation frequency corresponds to the top of the blue bar (i.e. C + RC). The error bars show the standard deviation in cooperation frequency computed over all initial conditions (for each parameter combination). On each panel, $\beta > (N - r_1)/r_1 r_2 (r_2 - 1)$ above the vertical bold line. Parameters: $N = 5$, $c_1 = 1$, $r_1 = 3$, $\alpha = 0.5$. 
Figure S19. Long-term average frequencies for the variant with rewards dependent on the PGG’s production and exclusion. Cooperation frequency corresponds to the top of the blue bar (i.e. C + RC). The error bars show the standard deviation in cooperation frequency computed over all initial conditions (for each parameter combination). On each panel, $\beta > (N - r_1)/r_1 r_2 (r_2 - 1)$ above the vertical bold line. Parameters: $N = 5$, $c_1 = 1$, $r_1 = 3$, $\alpha = 0.8$. 