Motion-boundary illusions and their regularization

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SUMMARY

Humans use various cues to understand the structure of the world from images. One such cue is the contours of an object formed by occlusion or from surface discontinuities. It is known that contours in the image of an object provide various amounts of information about the shape of the object in view, depending on assumptions that the observer makes. Another powerful cue is motion. The ability of the human visual system to discern structure from a motion stimulus is well known and has a solid theoretical and experimental foundation. However, when humans interpret a visual scene they use various cues to understand what they observe, and the interpretation comes from combining the information acquired from the various modules devoted to specific cues. In such an integration of modules it seems that each cue carries a different weight and importance. We performed several experiments where we made sure that the only cues available to the observer were contour and motion. It turns out that when humans combine information from contour and motion to reconstruct the shape of an object in view, if the results from the various modules devoted to specific cues are inconsistent, they experience a perceptual result which is due to the combination of the two modules, with the influence of the contour dominating, thus giving rise to the illusion. We describe here examples of such illusions and identify the conditions under which they happen. Finally, we introduce a computational theory for combining contour and motion using the theory of regularization. The theory explains such illusions and predicts many more. The same computational theory, when applied to retinal motion estimation, explains the effect of boundaries on the perception of motion that gives rise to a set of well-known illusions described by Wallach (1976).

1. THE THREE-DIMENSIONAL MOTION-BOUNDARY ILLUSION

If we display on a CRT screen the projection of a set of points that belong to the surface of a cylinder having a vertical axis, on the plane of the screen we produce a stimulus that does not provide much information to a human observer regarding the structure of the object in view (figure 1a). However, if the cylinder begins to rotate around its axis, then from the motion of the points on the screen the observer immediately perceives the rotating cylinder (Ullman 1979) (see figure 1b). This ability, well known as the kinetic depth effect, has been extensively studied. Computational theories, algorithms, implementations (Longuet-Higgins 1981; Longuet-Higgins & Prazdny 1980; Ullman 1979, 1981) and experimental work (Ullman 1979) suggest the existence of a process or a set of processes (modules in the sense of Marr [1982]) responsible for extracting the three-dimensional structure of an object in motion from its successive images. Research over the past 20 years has established various properties of this module (Ullman 1979; Marr 1982). However, the interaction between the modules of shape from contour and shape from motion, which creates some interesting perceptual phenomena, seems not to have been studied. Indeed, if we observe the rotating cylinder of figure 1b through the circular aperture of figure 1c (see figure 1d, which is the superposition of figure 1c on figure 1b), then we have the strong impression that we are observing a rotating ball (or sphere). In other words, although the motion of the points unambiguously suggests that the moving object is cylindrical, the presence of the circular occluding contour fools the visual system into reconstructing a spherical surface. It is well known (Ullman 1979) that given three orthographic projections of four non-coplanar points in a rigid configuration, the structure of these four points is uniquely determined (up to a Necker ambiguity). Similar results hold for perspective projection, where two views are actually adequate in general (Longuet-Higgins 1981; Longuet-Higgins & Prazdny 1980; Ullman 1981). In both cases (orthography or perspective) the problem of retinal correspondence first needs to be solved. This amounts to finding which feature in all the frames corresponds to the same feature in the world.

Similarly, if we view the rotating cylinder of figure 1b through the aperture of figure 1e, then we perceive a rotating vase (figure 1f). On the other hand, in these experiments the structure of the moving set of points does not seem to play the most important role. Indeed, if we view a rotating sphere, as in the experiment of figure 1d, through a rectangular aperture, then we perceive a rotating cylinder.

All the above-mentioned experiments are performed in a straightforward manner. The parameters of the
rotating object as well as the motion do not matter. What matters is only the spatial extent of the aperture and its position. For the illusions described here to be experienced, the aperture should not be small. If it is, then the perceived structure is planar. In addition, the position of the aperture is important in a non-obvious manner. For example, if the set of display points lie on a rotating sphere and the aperture is as in figure 1g, then a cone is perceived, but in figure 1h a planar surface is perceived. From the above experimental results it appears that when the human visual system needs to combine information from occluding contours and motion to reconstruct the scene in three dimensions, then for some contours and some surfaces it will experience an illusion, i.e. it will reconstruct (per-
2. A COMPUTATIONAL MODEL

First, the model assumes that the shape of the object – with regard to the observer – at the occluding boundaries is perpendicular to the contour and parallel to the image plane (under orthography) (figure 2c) or perpendicular to the plane defined by a small contour segment and the nodal point of the eye (under perspective). In the following, for simplicity and without loss of generality, we consider only orthographic projection. It is then worth noting that given two distinct orthographic projections of three points in a rigid configuration, the gradient \((p, q)\) of the plane that the three points define (with respect to the coordinate system of the first frame) lies on a conic section in gradient space. The coefficients of this conic section depend entirely on the interframe displacements of the points. Indeed, let the three points in space be \(O, A, B\) in their first position and \(O', A', B'\) in their second position, and let their projections in the two frames by \(O_1, A_1, B_1\) and \(O_2, A_2, B_2\), respectively. Also, let the gradient of the plane \(OAB\) be \(G = (p, q)\), and let

\[
\begin{align*}
O_1A_1 &= \alpha_1 = (x_1, y_1), \\
O_1B_1 &= \beta_1 = (x_2, y_2), \\
O_2A_2 &= \alpha_2 = (x_3, y_3), \\
O_2B_2 &= \beta_2 = (x_4, y_4).
\end{align*}
\]

Considering the geometry of the first projection \((OAB)\) to \(O_1A_1B_1\), and the second projection \((OAB)\) to \(O_2A_2B_2\), we have

\[
\begin{align*}
OA &= (x_1, y_1, G\alpha_1), \\
OB &= (x_2, y_2, G\beta_1), \\
OA' &= (x_3, y_3, G\alpha_2), \\
OB' &= (x_4, y_4, G\beta_2),
\end{align*}
\]

where \(\alpha, \beta, \gamma, \delta\) are to be determined.

However, because of rigid motion, we have \(\|OA\| = \|O'A'\|\), \(\|OB\| = \|O'B'\|\), where \(\|\cdot\|\) denotes length and \(\cdot\) dot product operation. Considering the above constraints with equations (5)–(8) we get

\[
\begin{align*}
\beta_1^2 - \beta_2^2 &= (G \cdot \alpha_1)^2 + (\alpha_1^2 - \alpha_2^2) (G \cdot \beta_1)^2, \\
-2(\alpha_1 \beta_1 - \alpha_2 \beta_2) (G \cdot \alpha_1)(G \cdot \beta_1), \\
(\alpha_1^2 - \alpha_2^2)(\beta_1^2 - \beta_2^2) - (\alpha_1 \beta_1 - \alpha_2 \beta_2)^2 &= 0.
\end{align*}
\]

Given that \(G \cdot \alpha_1 = px_1 + qy_1\) and \(G \cdot \beta_1 = px_1 + qy_1\), (10) is of the form

\[
\alpha^2 + \beta^2 + \gamma \alpha \beta + \delta = 0,
\]

where the coefficients \(\alpha, \beta, \gamma, \delta\) depend on the image vectors \(\alpha_1, \alpha_2, \beta_1, \beta_2\). The above relation indicates that given two projections of a set of points in a rigid configuration, the gradient of the surface is locally constrained by the retinal motion to lie on a conic section in gradient space. Also, it is clear that if several views are available, then from the intersection of all the conics a unique solution for the local gradient \((p, q)\) is determined (unique up to the Necker reflection \((-p, -q)\)). However, here we desire to find that surfaces which best satisfies the contour constraints, the motion constraints, and is as smooth as possible; we have assumed that we only allow smooth surfaces. Before we proceed, we need to change the coordinate system expressing the gradient \((p, q)\) to a more convenient form that can handle orientation at occluding boundaries.

Such a convenient system is given by the stereographic coordinates (see Horn (1986)). The relation between the stereographic coordinates \((f, g)\) and the gradient \((p, q)\) is given by:

\[
\begin{align*}
f &= 2p[\sqrt{(1 + p^2 + q^2) - 1}]/(p^2 + q^2); \\
g &= 2q[\sqrt{(1 + p^2 + q^2) - 1}]/(p^2 + q^2).
\end{align*}
\]

Let \(L(f, g, x, y) = 0\) represent the motion constraint (equation 10) in the new coordinates. If \(D\) is the boundary, we wish to recover that surface \((f(x, y), g(x, y))\) \((x, y)\) inside \(D\) which satisfies the boundary constraints (if \(i \in D\) then \(f_i^2 + g_i^2 = 4\)), as smooth as possible, and at the same time best satisfies the constraint \(L(f, g, x, y) = 0\) everywhere on the image. Following the paradigm introduced by Poggio et al. (1985), we find that such a surface is the one that minimizes the functional

\[
e = \int \frac{1}{2} ((f_x)^2 + (f_y)^2 + (g_x)^2 + (g_y)^2) + \lambda(L)^2 \, dx \, dy,
\]

where the subscripts denote partial differentiation and \(\lambda\) is the regularization parameter, weighing the relative importance of the two factors comprising the above energy functional that needs to be minimized. It was shown by Aloimonos & Brown (1989) that there exists a unique global minimum of \(e\) under an appropriate choice of \(\lambda\). Here, we need to emphasize that according to this formulation of the optimization there is no guarantee that the resulting surface will be integrable \((f_x = g_z, g_x = f_z)\). Thus, the minimization should really be subject to his constraint. However, for the cases considered here, the lack of an integrability condition does not affect the problem, as seen in the results. If we discretize the problem and consider summations instead of integrals and differences instead of derivatives, then \(e\) is written as

\[
e = \sum_{i,j} (S_{ij} + \Lambda_{ij}),
\]

where

\[
S_{ij} = (1/m^2)[(F_{i+1,j} - F_{ij})^2 + (F_{ij-1} - F_{ij})^2]
\]

and where \(F_{i,j}, g_{i,j}\) represent the surface orientation at the regular grid point \((im, jm)\). This minimization is subject to boundary conditions, i.e. \(F_{i,j}\) and \(g_{i,j}\) are known if \((im, jm)\) belong to the boundary. We assume that the surface normal at a boundary point \((ij)\) is parallel to the image plane (i.e. \(F^2 + m^2 = 4\)). (Occluding boundary.)

Function \(e\) is defined on a compact subset \(K\) of \(R^{2n}\) for some \(n\) and it is continuous with respect to \(F_{i,j}\) and \(g_{i,j}\). Therefore, there exists a solution to the minimization problem. Furthermore, the solution that
minimizes $\varepsilon$ is the solution of the system (Euler-Lagrange equations):

$$\frac{\partial \varepsilon}{\partial f_{i,j}} = \frac{\partial \varepsilon}{\partial g_{i,j}} = 0.$$  

Equations (15) become

$$f_{i,j} = f_{i,j}^* - \frac{1}{2} \lambda m^2 \left\{ L(f_{i,j}, g_{i,j}; i,j) \right\} \frac{\partial L / \partial f}{\partial f_{i,j}} \left\{ f_{i,j}, g_{i,j}; i,j \right\};$$

$$g_{i,j} = g_{i,j}^* - \frac{1}{2} \lambda m^2 \left\{ L(f_{i,j}, g_{i,j}; i,j) \right\} \frac{\partial L / \partial g}{\partial g_{i,j}} \left\{ f_{i,j}, g_{i,j}; i,j \right\},$$  

where

$$f_{i,j}^* / g_{i,j}^* = \frac{1}{2} \left\{ f_{i+1,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1} + f_{i+1,j-1} \right\}.$$  

Equations (16) define an iteration that provides the solution (global minimum of $\varepsilon$) (Aloimonos & Brown 1989).

To summarize, the mathematical model that we have introduced reconstructs, from occluding contour and local motion information, the surface that is as smooth as possible and best satisfies the motion constraint everywhere, while satisfying the boundary conditions provided by the form of the contour; it is found as the limit of the iterations (16). Using these iterations, we reconstructed the surfaces giving rise to the displacement fields of figure 3a, b. They both come from the rotation (with velocity $\omega_x = 1$, $\omega_y = 2$ and $\omega_z = 3$) of a cylinder with a vertical axis, and the second one is viewed through a triangular aperture. Using iterations (16) in each case and utilizing the appropriate boundary conditions, we recovered the surfaces of figures 3c and 3d respectively, which are consistent with human perception. The results of applying the algorithm to all other illusion-producing inputs described earlier are also consistent with human perception. It is thus clear that when humans combine the cues of occluding contours and local motion to reconstruct the moving object in view, then they reconstruct (see) the surface that is consistent with the boundaries, is as smooth as possible, and best satisfies the local motion constraints. The mathematical framework of regularization is adequate for describing the perceptual process that combines the cues of motion and contour for the purposes of visual reconstruction.

3. THE TWO-DIMENSIONAL MOTION-BOUNDARY (OR WALLACH) ILLUSSIONS

At this point it is interesting to note that the same theory can be employed for describing the perceptual filter responsible for the estimation of retinal motion in some cases. Indeed, when humans observe through a specific aperture of moving pattern whose structure is such that only the component of image motion (flow) along the direction of the gradient of the image intensity function can be estimated, then the perceived motion direction depends entirely on the shape of the aperture. Wallach (1976) produced a series of such striking illusions. These illusions appear because the visual system needs to combine two sets of measurements: measurements of optical flow along the aperture boundaries and measurements of normal flow inside the aperture. There are psychophysical and physiological data (Adelson & Movshon 1982) suggesting that motion is computed in two stages in the primate visual cortex. In the first stage, the normal components of the velocity are computed and in the second stage these measurements are combined over an extended area into a coherent motion pattern. The process of combining various observations into a coherent pattern may be achieved through several mathematical frameworks such as regularization (Poggio et al. 1985; Poggio
be combined into a coherent pattern, under the assumption that the values at the boundaries of the aperture will be as initially computed. One way to enforce coherence in the finally estimated flow field would be to look for the smoothest flow field subject to the normal flow constraints and the boundary constraints. As before, one may require that the optical flow field \((u(x,y),v(x,y))\) \(\forall x\) inside \(D\), where \(D\) is the aperture, is as smooth as possible (Poggio & Koch 1985; Aloimonos & Shulman 1987; Aloimonos & Brown 1989). If the estimated normal optic flow at every point \((x,y)\) is \((u_n(x,y),v_n(x,y))\) then a coherent flow is the minimizer of the functional

\[
e(u,v) = \int_D \{L(x,y) + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)\}\,dx\,dy,
\]

where \(L = [u(x,y) - u_n(x,y)]^2 + [v(x,y) - v_n(x,y)]^2\) and \(\lambda\) is a parameter called the regularization constant. Note that the above functional consists of two terms: the first term denotes the consistency of the solution (flow field) with the data, and the second term denotes smoothness. The parameter \(\lambda\) is related to the relative importance of the constraint versus smoothness, as before. Thus, by finding the flow field that minimizes \(e\), we find the flow that is as smooth as possible, and at the same time is as close as possible to the initially computed values. To obtain a solution to this minimization problem, replace integrals by summations and derivatives by differences in order to discretize the problem. If \(D\) represents the image (see figure 4a), then the flow \((u_{ij},v_{ij})\) at each grid point \((i,j)\) is the minimizer of

\[
e = \sum_{i,j} ([u_{ij} - u_{ij}^n]^2 + [v_{ij} - v_{ij}^n]^2 + \lambda S_{ij}],
\]

where

\[
S_{ij} = \frac{1}{4}[(u_{i,j+1} - u_{ij})^2 + (u_{i+1,j} - u_{ij})^2 + (v_{i,j+1} - v_{ij})^2 + (v_{i+1,j} - v_{ij})^2]
\]

is the normal flow (computed) at a point \((i,j)\). As before the minimization of \(e\) is achieved through the iteration

\[
u_{ij}^{(o+1)} = \nu_{ij}^{(o)} - \frac{\lambda L(i,j)}{\partial L(i,j)/\partial u_{ij}^{(o)}};
\]

\[
v_{ij}^{(o+1)} = \nu_{ij}^{(o)} - \frac{\lambda L(i,j)}{\partial L(i,j)/\partial v_{ij}^{(o)}},
\]

where \(o\) denotes the iteration number and

\[
u_{ij}^{(o+1)} = \frac{1}{4}[(u_{i+1,j} + v_{i+1,j} - u_{i,j+1} - v_{i,j+1}) + (u_{i+1,j} - v_{i+1,j} + u_{i,j+1} - v_{i,j+1}) + u_{i+1,j}(v_{i,j+1}) + u_{i,j+1}(v_{i+1,j})].
\]

By using the iterative process described above, we can find the image motion in the Wallach illusions; it turns out that in all cases the results are consistent with human perception. The important factor is always the values of the flow at the boundaries; because they are not allowed to change during the iteration process, consequently they drive the whole process. Indeed, consider figure 5. In 5a, the grating is moving horizontally. The values at the boundaries are as shown. The normal flow inside the aperture is again as shown. Using these initial values, the iterative formulae (17) and (18) computed the flow field of 5b, which is consistent with perception. Performing a similar ex-

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Figure 4. (a) A set of parallel oblique lines moving behind a rectangular aperture. (b) An obliquely oriented pattern moving behind a rectangular aperture. (c) A set of parallel lines moving behind a triangular aperture. (d) A set of parallel oblique lines moving behind a circular aperture.

& Torre 1984; Poggio & Koch 1984) or its variants (Aloimonos & Shulman 1987), Markov random fields, fuzzy sets and information compression methodologies (Aloimonos & Shulman 1989). We show here that if regularization theory is used by a system for combining various motion estimates into a coherent pattern, then that system will experience the Wallach aperture illusions. This, in conjunction with the previously described results and other results from the literature (Büthoff et al. 1987) predicting the motion capture illusions, the barber pole illusion and others, using algorithms employing regularizing operators, suggests that some form of regularization is performed by cortical connections in the circuitry of the primate visual cortex responsible for retinal motion estimation.

If we observe an obliquely oriented grating (or a set of parallel lines) (figure 4a, b) moving horizontally behind a vertical slit, we see vertical motion. If we observe the same pattern moving vertically behind a horizontal slit, we see horizontal motion. Actually, the illusion is experienced for any motion of the grating. In the case of the vertical aperture, the illusion will happen for any velocity between 0° and 180°. If the estimated normal optic flow at every point \((x,y)\) is \((u_n(x,y),v_n(x,y))\) then a coherent flow is the minimizer of the functional

\[
e(u,v) = \int_D \{L(x,y) + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)\}\,dx\,dy,
\]

where \(L = [u(x,y) - u_n(x,y)]^2 + [v(x,y) - v_n(x,y)]^2\) and \(\lambda\) is a parameter called the regularization constant. Note that the above functional consists of two terms: the first term denotes the consistency of the solution (flow field) with the data, and the second term denotes smoothness. The parameter \(\lambda\) is related to the relative importance of the constraint versus smoothness, as before. Thus, by finding the flow field that minimizes \(e\), we find the flow that is as smooth as possible, and at the same time is as close as possible to the initially computed values. To obtain a solution to this minimization problem, replace integrals by summations and derivatives by differences in order to discretize the problem. If \(D\) represents the image (see figure 4a), then the flow \((u_{ij},v_{ij})\) at each grid point \((i,j)\) is the minimizer of

\[
e = \sum_{i,j} ([u_{ij} - u_{ij}^n]^2 + [v_{ij} - v_{ij}^n]^2 + \lambda S_{ij}],
\]

where

\[
S_{ij} = \frac{1}{4}[(u_{i+1,j} - u_{ij})^2 + (u_{i,j+1} - u_{ij})^2 + (v_{i+1,j} - v_{ij})^2 + (v_{i,j+1} - v_{ij})^2]
\]

is the normal flow (computed) at a point \((i,j)\). As before the minimization of \(e\) is achieved through the iteration

\[
u_{ij}^{(o+1)} = \nu_{ij}^{(o)} - \frac{\lambda L(i,j)}{\partial L(i,j)/\partial u_{ij}^{(o)}};
\]

\[
v_{ij}^{(o+1)} = \nu_{ij}^{(o)} - \frac{\lambda L(i,j)}{\partial L(i,j)/\partial v_{ij}^{(o)}},
\]

where \(o\) denotes the iteration number and

\[
u_{ij}^{(o+1)} = \frac{1}{4}[(u_{i+1,j} + v_{i+1,j} - u_{i,j+1} - v_{i,j+1}) + (u_{i+1,j} - v_{i+1,j} + u_{i,j+1} - v_{i,j+1}) + u_{i+1,j}(v_{i,j+1}) + u_{i,j+1}(v_{i+1,j})].
\]

By using the iterative process described above, we can find the image motion in the Wallach illusions; it turns out that in all cases the results are consistent with human perception. The important factor is always the values of the flow at the boundaries; because they are not allowed to change during the iteration process, consequently they drive the whole process. Indeed, consider figure 5. In 5a, the grating is moving horizontally. The values at the boundaries are as shown. The normal flow inside the aperture is again as shown. Using these initial values, the iterative formulae (17) and (18) computed the flow field of 5b, which is consistent with perception. Performing a similar ex-
experiment (figure 5c) where the aperture is a horizontal slit, we find the (mostly) horizontal flow field of figure 5d.

If a set of parallel lines is moving behind a triangular aperture, as reported by Wallach (1976), various motion processes are observed. Different parts of the pattern appear to move simultaneously in different directions. The lines appear to move parallel to edge A, parallel to B, and parallel to C (Figure 4c). This is consistent with the flow field of figure 5e that the iteration of (17) and (18) converged to from the initial values and boundary conditions of figure 5f. If the height of the triangle increases, then the moving pattern seems to split in two. The lines adjacent to edge B move in the direction of that edge, whereas the lines that end at edge C seem to move parallel to C. This is again consistent with the results of our iteration (figure 5g). Similarly, if the aperture is a circle, our iteration provides the flow field of figure 5h with initial conditions the ones of figure 5i. This is consistent with perception.

The parameter \( \lambda \) involved in the iteration described above (which iteration basically solves the linear system \( \frac{\partial e}{\partial u_i} = 0, \frac{\partial e}{\partial u_j} = 0, \forall i, j \in D \)) weighs the relative importance of the constraints (closeness to the normal flow values) versus the smoothness. In our experiments \( \lambda \) was taken larger than 50. It is however clear that if \( \lambda \) is large enough, then what we actually find is very smooth flow field consistent with boundary the conditions. If \( \lambda \) is very large, the requirement that the solution is close to initial data (normal component) – the first term of functional \( e \) – lessens considerably. The reason that in the case of the vertical slit the horizontal motion produces the illusion of vertical motion (figure 5a) is that the vertical dimension \( v \) is larger than the horizontal one \( v > h \). That means that there are many more vertical vectors at the boundaries than horizontal ones. Because the boundary values are not allowed to change, the iterative process that attempts to find a smooth flow field consistent with the boundaries will be very much influenced by the vertical boundary values. In other words, one can think of the process as a controlled propagation of the boundary values. The vertical boundary values are many and smoothness will make most of the flow values "similar" to them, i.e. vertical. Thus, the impression of vertical motion. If \( v < h \), then the opposite happens, because the horizontal boundary values dominate. And if \( v = h \), then the contribution of both horizontal and vertical boundary values is equal; consequently the estimated motion is along the diagonal (see figure 5j) and this is consistent with perception.

It should be emphasized here that a large variety of algorithms for the estimation of optic flow (see (Horn (1986) for a survey) will produce results consistent with perception when confronted with Wallach-like inputs. The reason that this happens is of course that all these algorithms have inside them some form of regularization (smoothing, averaging, etc.). And what was described is not an algorithm for the estimation of optic flow; it is just a description of the principle of regularization as applied to combining flow estimates into a coherent pattern.

Based on the above experimental and theoretical results, one cannot reject the hypothesis that cortical connections in the primate visual cortex implement some form of regularization for motion perception, in both the estimation of retinal motion and its interpretation for the purposes of reconstruction. It was suggested by Poggio et al. (1985) that the theory of regularization may be used as a framework for low-level vision. The present work demonstrates that it is beneficial to think in this paradigm.

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